STATISTICAL COMPARISON OF ELECTRICAL EFFICIENCY THEORY TO ARCHIE'S EQUATIONS AND EFFECTIVE MEDIUM THEORY

ABSTRACT

A comparison is made between Archie, electrical efficiency theory, and effective medium theory in both rocks with both conductive and nonconductive matrix. It is found that the as-yet undefined theory for electrical efficiency is not needed, because Archie and effective medium theory are as good if not better than the electrical efficiency equations at predicting water saturation in porous rocks.

INTRODUCTION

Herrick and Kennedy (1993, 1994, and 2009) developed a conceptual framework for incorporating tortuosity into rock conductivity theory. Their variable electrical efficiency *(E)* is plotted against either porosity (ϕ) or the product of water saturation (S_w) and ϕ to compare bulk rock conductivity to that of a cylindrical tube. By studying the behavior of *E*, the authors expect that a first-principle theory of rock conductivity will eventually be derived to explain the apparent linearity of *E*. Also, they believe that this new theory, called "electrical efficiency theory" (EET), will eventually replace Archie's (1942) equations in formation evaluation. However, the EET variables as defined are all regression coefficients.

On the other hand, Archie's porosity-based equations can be derived directly from effective medium theory (EMT) by setting matrix conductivity (C_r) to zero. EMT has been used in the study of rock conductivity since at least the early 1980's; a recent review of EMT can be found in Berg, 2007.

A BRIEF REVIEW OF THE MODELS STUDIED

Electrical efficiency theory

EET was developed to model the properties of both fully and partially water-saturated rocks. In the theory, 100% water-saturated electrical efficiency (E_0) represents the difference in behavior between a water-filled tube and a same-sized cylinder of water-saturated rock. Similarly, the partially saturated electrical efficiency variable (E_t) represents the difference between a partially water-filled tube and a same-size cylinder of partially water-saturated rock. Following is the saturation relationship for electrical efficiency:

$$E = \frac{C_t}{C_w S_w \phi} \quad (1)$$

where

E_t = partial-S_w electrical efficiency,

C_t = partial-S_w whole-rock conductivity, and

C_w = water conductivity.

At 100% S_w, equation (1) reduces to

$$E_0 = \frac{C_0}{C_w \phi} \quad (2)$$

where

E_0 = electrical efficiency at 100% S_w, and

C_0 = rock conductivity at 100% S_w.

On the basis of the apparent linearity of plots of electrical efficiency from laboratory measurements, the following expressions for analyzing rock conductivity and water saturation were given:

$$E_0 = a_0\,\phi + b_0, \quad (3)$$

$$E_t = a_t\,(S_w \cdot \phi) + b_t, \quad (4)$$

where a_t, b_t, a_0, and b_0 are regression coefficients.

Archie's law and saturation equation

Archie's law (1942, equation 3) formulated for 100% water saturated rock is as follows:

$$\phi^m = \frac{C_0}{C_w} \tag{5}$$

where

m = cementation or porosity exponent.

This relationship has been the dominant relationship for studying conductivity in clean, porous rocks ever since it was published. (The curve-fitting variable *a*, which is sometimes incorrectly attributed to Archie, is not used here.)

Following is Archie's porosity saturation equation (1942, combined equations 3 and 4) in terms of conductivity:

$$S_w{}^n \phi^m = \frac{C_t}{C_w}, \tag{6}$$

where

n = saturation exponent.

This equation has been used extensively for S_w calculations on clean formations.

Effective medium theory

Although Archie's equations (5 and 6) were formulated empirically, effective medium theory provides theoretical bases for m and n (see Sen, et al. 1981 and Bussian, 1983). The effective medium theory discussed here was developed by Hanai (1960a, 1960b) from theory originated by Bruggeman (1935) to predict the electrical properties of oil-in-water and water-in-oil emulsions. This relationship has been shown by Bussian (1983), Sen et al. (1981) and others, to be capable of modeling the electrical properties of 100% water-saturated rocks. Following is Bussian's (1983, equation 7) adaptation of the HB equation to the study of rocks:

$$\phi = \left(\frac{C_w}{C_0}\right)^{\frac{m-1}{m}} \left(\frac{C_0 - C_r}{C_w - C_r}\right) \tag{7}$$

where

C_r = matrix conductivity.

When C_r is set to zero, equation 7 reduces to Archie's law (equation 5). Archie's S_w equation (6) can be derived by substituting hydrocarbon/water fluid conductivity based on equation 5 back into the actual equation 5. This is a "hydrocarbons first" derivation. For more background on this derivation and on the use of effective medium theory for calculation of S_w, refer to Berg (2007).

COMPARISONS

Nonconductive matrix

In order to assess the EET vs. Archie, least-squares fits of the two approaches were constructed using the same data sets those used in the figures by Herrick and Kennedy (Figure 8, 1993; Figure 7, 1994; and Figure 7, 2009). Figure 1 displays the results. Visually, it is difficult to determine which is best, so the numerical results have been listed in Table 1. The average difference between EET and Archie correlation coefficients is negligible.

There is a simple explanation for the seeming linearity of E versus $S_w\phi$. Almost any nonlinear relationship, if viewed over a small enough interval, will appear linear. The range of measured $S_w\phi$ with respect to the total rock volume can never be more than ϕ and is usually substantially less, which makes it no surprise that the plots appear linear.

Conductive matrix

Waxman and Smits' (1968) experimental data set (published in Clavier, et al., 1984) was used to compare EET and EMT in rocks with conductive matrix. Figure 2 shows an EET fit using equation 11 from Herrick and Kennedy (1993) and an EMT fit using the algorithm in Berg (2007). Although EET had 4 free variables and EMT had only 3 free variables, the EMT fit is considerably better. For all 12 samples in the Waxman and Smits data set, the average correlation coefficient squared (r^2) for EET was

0.9739, while r^2 for EMT was 0.9870. In all cases, the EET value for r^2 was lower than the EMT value. All fits were performed using the downhill simplex technique, and on samples 3279B and 521C, the Microsoft® Excel "Solver" add-in was used to double-check the simplex calculations.

CONCLUSION

Since Archie/EMT works equally as well or better than EET on experimental data, it appears that a new theory is not warranted. As of this writing, no physical meaning has been given for the EET regression coefficients. On the other hand, the meaning of m, n, and C_r are well known, if not universally accepted. Until such time as physical meaning is assigned to the EET coefficients and a true theory is formulated, it appears that EET remains a regression approach.

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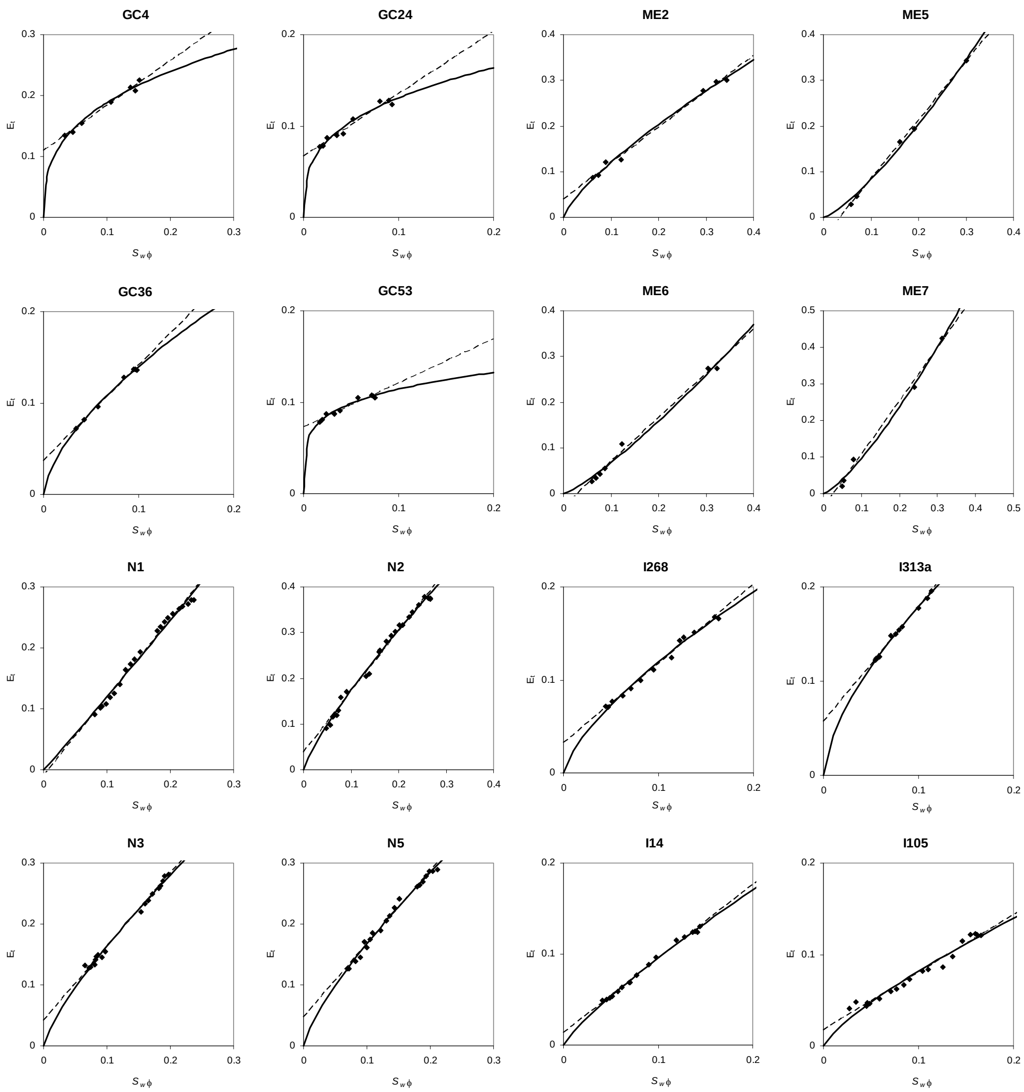

Figure 1. Curve fits for the 16 samples from Herrick and Kennedy (Figure 8, 1993; Figure 7, 1994; and Figure 7, 2009.) The EET fits are dashed lines, while the Archie fits are solid curves. The curve fits for Archie are visually just as good as the curve fits of EET. (Although Herrick and Kennedy, 2009, state that there is only one adjustable parameter in their equation, their curve fits always use both coefficients.) All calculations were done using Microsoft® Excel and checked using the downhill simplex technique.

Table 1. Calculations from the EET and Archie curve fits (Figure 1) to the data from Herrick and Kennedy (1993, 1994, and 2009). An r^2 of 1 is a perfect correlation.

Sample	EET a_t	EET b_t	EET r^2	Archie m	Archie n	Archie r^2	EET - Archie r^2
GC4	0.7360	0.1094	0.9876	1.8069	1.3471	0.9783	0.0093
GC24	0.6830	0.0666	0.9621	1.8664	1.3236	0.9648	-0.0027
GC36	1.0425	0.0367	0.9962	1.8520	1.6369	0.9968	-0.0006
GC53	0.4819	0.0725	0.9477	1.8610	1.2128	0.9612	-0.0134
ME2	0.7858	0.0391	0.9930	2.1069	1.7590	0.9926	0.0004
ME5	1.2834	-0.0452	0.9990	1.8822	2.3006	0.9955	0.0035
ME6	0.9678	-0.0283	0.9921	2.1128	2.2245	0.9875	0.0046
ME7	1.4583	-0.0401	0.9930	1.7411	2.2980	0.9925	0.0005
N1	1.2767	-0.0097	0.9865	1.8544	2.0362	0.9857	0.0009
N2	1.3254	0.0376	0.9887	1.7204	1.8149	0.9924	-0.0037
N3	1.2041	0.0417	0.9940	1.7895	1.7777	0.9917	0.0024
N5	1.1988	0.0467	0.9877	1.7807	1.7664	0.9912	-0.0035
I313	0.6869	0.0956	0.8122	1.8871	1.4201	0.8771	-0.0649
I313a	1.2227	0.0565	0.9941	1.7831	1.6313	0.9956	-0.0015
I105	0.6299	0.0173	0.9736	2.1770	1.7736	0.9610	0.0126
I268	0.8540	0.0319	0.9909	1.9810	1.7085	0.9891	0.0018
I14	0.8137	0.0133	0.9951	2.0539	1.8373	0.9955	-0.0004
Average	0.9795	0.0319	0.9761	1.8974	1.7570	0.9793	-0.0032

Note: Sample I313 contains all experimental points, while Sample I313a has points omitted to be consistent with the Herrick and Kennedy papers.

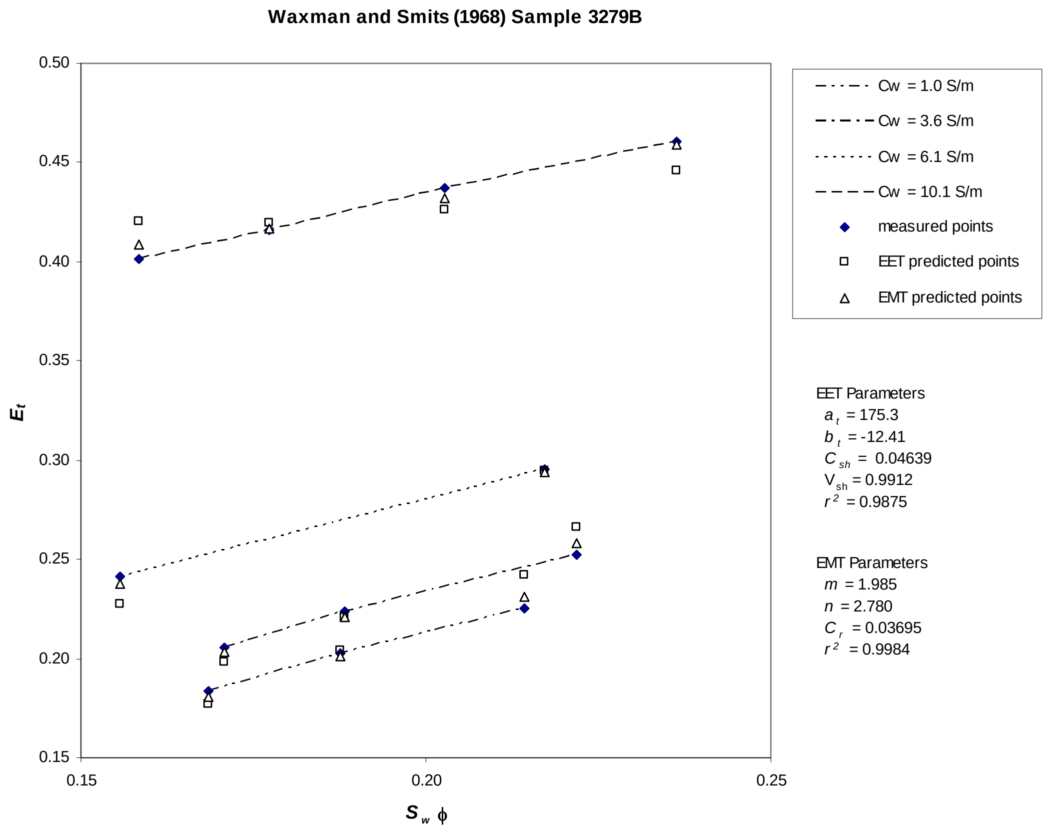

Figure 2. EET and EMT least-squares fits to Waxman and Smits (1968) sample 3279B. Since the experiment used 4 values of C_w, the 4 trend lines are shown for clarity. However, these are not multiple fits, but two single fits done with multiple variables. In other words, in each fit, all of the points of the sample were minimized with respect to least squares. The appearance of multiple curve fits is because the experiment was done with multiple values of C_w, and in both relationships, E is dependent on C_w. Note: EET used 4 free variables (a_t, b_t, C_{sh}, and V_{sh}), while EMT used 3 free variables (m, n, and C_r).