

CALCULATING TRUE STRATIGRAPHIC AND TRUE VERTICAL THICKNESS

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Standard practice for calculating true stratigraphic thickness (*TST*) and true vertical thickness (*TVT*) traditionally involves a single dip, a single deviation measurement and a measured-depth interval (Figure 1). The usual procedure is to plug dip, dip azimuth, borehole inclination, and borehole azimuth of the top of a bed, along with the measured depth interval, into the standard equation (Tearpock and Bischke, 1991). Since dip and deviation are usually different at the top and bottom of a given interval, how do we adjust our calculations accordingly? This paper will resolve that issue by first putting the problem into vectoral form. The two-deviation problem is easily solved by using spatial coordinates derived from borehole deviation calculation in a vectoral equation. To solve the two-dip problem, vector averaging is used to calculate an averaged dip to substitute into the vectoral formula.

TST is normally calculated from *TVT* by dividing by the cosine of dip. However, since *TVT* conceptually is derived from *TST*, mainly *TST* will be discussed here. In any case, *TVT* can always be calculated from *TST* by dividing by the cosine of the bedding dip.

Two recent papers have introduced two-dip models for calculating *TST*, Xu, et al., 2007 and 2010. Both papers implicitly assume that the upper and lower dips have the same azimuth. Therefore, they are not fully 3D methods and will not be discussed here.

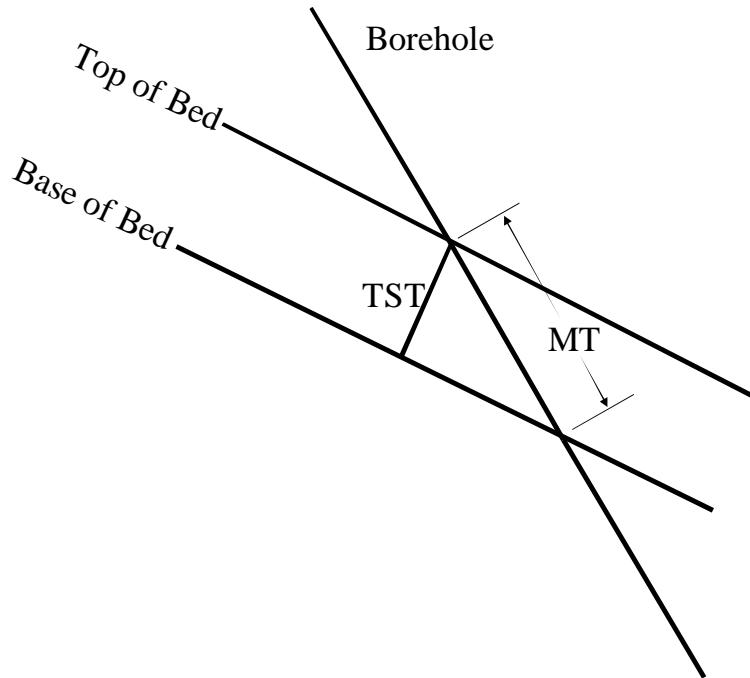


Figure 1. Diagram showing the configuration of the borehole and a bed whose TST is being calculated. (MT is the difference in measured depth.) The plane of section does not have to be vertical, but the borehole and the dip normal do have to lie within it. This is a fully three-dimensional technique.

Following is the formula from Tearpock and Bischke (1991), modified somewhat to be consistent with the vectoral derivation later on:

$$TST = MT \cdot (\cos \phi_2 - \sin \phi_2 \cdot \cos(\theta_1 - \theta_2) \cdot \tan \phi_1) \cdot \cos \phi_1, \quad (1)$$

where MT = measured thickness, TST = true stratigraphic thickness, ϕ_1 = dip, θ_1 = the dip azimuth, ϕ_2 = borehole inclination from vertical, and θ_2 = borehole azimuth. To calculate TVT , TST is divided by the cosine of the dip or

$$TVT = \frac{TST}{\cos \phi_1}. \quad (2)$$

There are several published methods that will produce identical results, but only the method described in Tearpock and Bischke (1991) will be discussed here. In addition, discussion involving TST calculation will apply as well to TVT calculation, since $TVT = TST/\cos\phi_1$. Equation 1 will be referred to as the geometric single-dip equation.

Treating *TST* and *MT* as Vectors

The Single-Dip Vectorial Equation

Figure 2 shows Figure 1 in vectoral form. The vectors \vec{d} and \vec{b} represent the scalars *TST* and *MT*, while α is the angle between the vectors. From Figure 1 it can be seen that

$$TST = MT \cdot \cos \alpha . \quad (3)$$

Since the cosine of the angle between two vectors is the dot product, equation 3 in vectoral notation is

$$TST = MT \cdot \hat{d} \bullet \hat{b} , \quad (4)$$

where \hat{d} is the dip direction and \hat{b} is the borehole direction.

The geometric single-dip equation (1) can be derived symbolically from equation 4 using the spherical to rectangular conversions below:

$$x = \sin \phi \cdot \sin \theta , \quad (5)$$

$$y = \sin \phi \cdot \cos \theta , \text{ and} \quad (6)$$

$$z = -\cos \phi , \quad (7)$$

where x , y , and z are the rectangular coordinates of the vector, ϕ is the polar angle, and θ is the azimuthal angle. This means that the geometric single-dip equation (1) is equivalent to the vectoral single-dip equation (8) below. (In this particular coordinate system, positive x is east, positive y is north, and positive z is upward.)

Returning to the vectoral derivation, since the borehole vector \vec{b} is equal to $MT \cdot \hat{b}$, equation 4 can be written

$$TST = \hat{d} \bullet \vec{b} . \quad (8)$$

In terms of the vector components, equation 8 is

$$TST = x_d X_b + y_d Y_b + z_d Z_b , \quad (9)$$

where x_d , y_d , and z_d are the direction cosines of the lower hemisphere dip pole and X_b , Y_b , and Z_b are the spatial coordinates of bottom point relative to the top point. Equation 9 is

the vectoral single-dip equation. Combining equations 2 and 7, the vectoral equation for TVT is

$$TVT = \frac{TST}{-z_b}. \quad (10)$$

It is assumed here that the deviation calculation that is being done is based on the minimum curvature method (Taylor and Mason, 1972). Even if that is not the case, any good deviation calculation method should be able to provide rectangular coordinates suitable for calculation of TST . However, other deviation-calculation methods that use techniques such as spline fits may not be amenable to interpolation techniques discussed below.

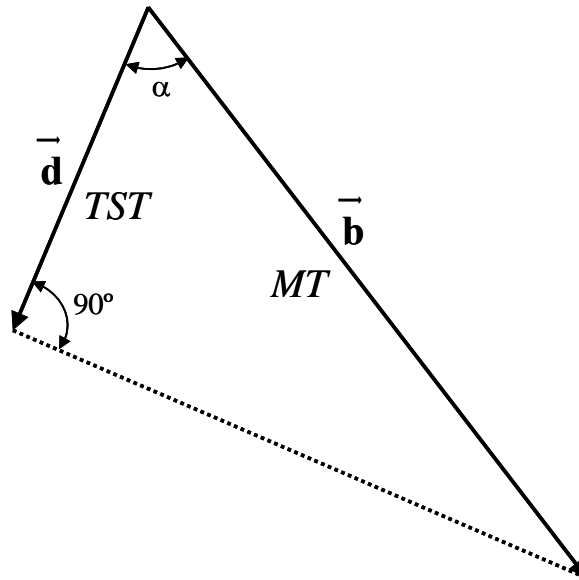


Figure 2. The vectoral equivalent to Figure 1. As with their TST and MT in Figure 1, vectors \vec{d} and \vec{b} both lay within a plane, but that plane is not necessarily vertical.

Equation 9 automatically honors both top and bottom deviations, because both are used in the calculation of the deviation coordinates. In addition, the straight-line distance MT is used, as opposed to traditional methodology which simply uses the difference in measured depth. Figure 10 shows a borehole superimposed over the vectors shown in Figure 2. It is clear that the difference in measured depth will be larger than the straight-

line distance MT . Although significant, the error caused by using measured depth can be small compared to the error caused by using only one of the dips.

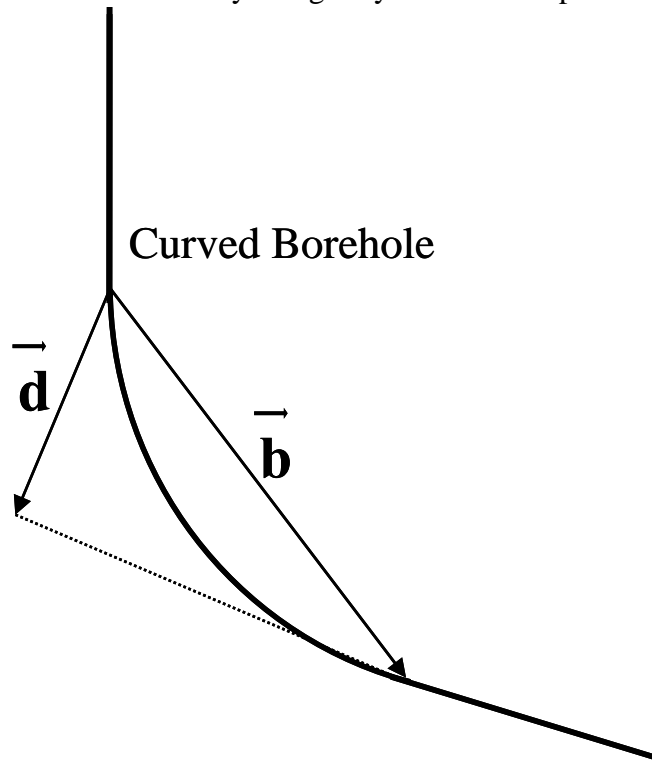


Figure 3. The configuration in Figure 2 with a highly deviated borehole. In this example it is clear that simply taking the difference in measured depth of the bottom and top of the bed will not give the correct length for MT and therefore TST . Furthermore, using either deviation by itself is totally inadequate for calculating TST .

Using Figure 3 for an approximate configuration, assume that everything is in the same, vertical plane of section. Using a measured-depth interval of 500m, dip of 23° , top inclination of 0° , and bottom inclination of 73° , MT calculates at 467m. Using that value for MT results in TST of 237m, while the result from using measured depth is 254m. Furthermore, if the calculation uses only the top deviation, as is customary, calculated TST is 460m, nearly twice the actual value. Most surprising, if only the lower deviation value is used, the TST comes out to -52m, which initially seems wrong. It is not wrong,

however, because the borehole in such a configuration would actually be traveling slightly up section and TST is negative when the borehole is traveling up section.

The Vector Average Method

The vector average method expands on the vectoral equation derived above by replacing the single dip vector with a vector representing the average of the upper and lower dips. Figure 4 shows the configuration of the vector average method for finding TST . The simplest way to calculate the vector average \hat{d} is to normalize the vector sum:

$$\hat{d} = \frac{\hat{d}_1 + \hat{d}_2}{|\hat{d}_1 + \hat{d}_2|}. \quad (11)$$

Substituting equation 11 into equation 8 yields the vector average equation

$$TST = \frac{\hat{d}_1 + \hat{d}_2}{|\hat{d}_1 + \hat{d}_2|} \bullet \vec{b}, \quad (12)$$

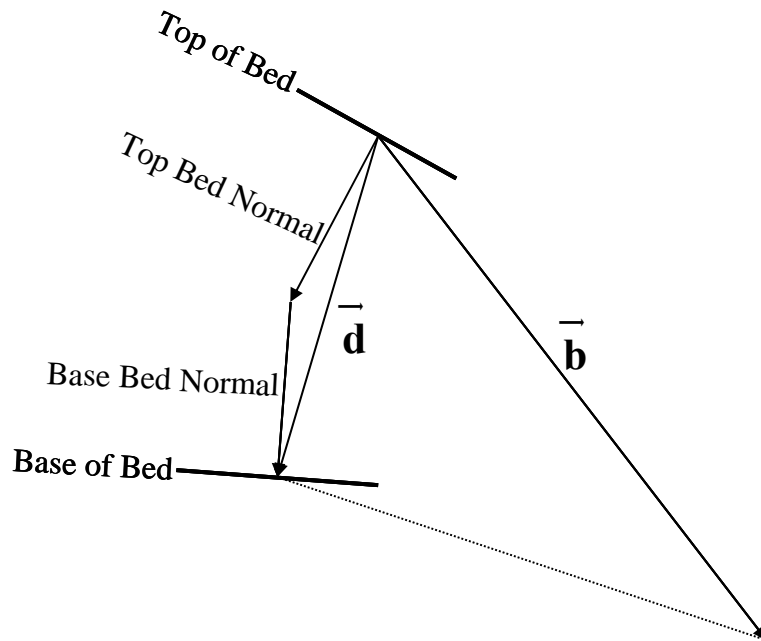


Figure 4. The vector average method for incorporating both top and bottom dips. Since only the direction \hat{d} is needed in equation 8, it can be found as the normalized sum of the two dip-pole vectors.

The vector average method is an approximation because it does not have an underlying geometric basis. However, it is fully 3D and for small changes in dip, which should nearly always be the case, it is fairly accurate. Work is currently being done to derive an exact equation that is equivalent to the incremental method described below.

The Incremental Method

This method is the most accurate two-dip method for calculating *TST*. New dips and deviations are interpolated between the top and bottom of an interval and then successive *TST* calculations are summed for the *TST* of the interval. Interpolation methods are discussed in the interpolation section below. Both this method and the vector average method have out-of-plane and geometrical effects that make calculation of *TVT* from them problematic, and therefore equation 2 is not valid for them.

Discussion

Interpolation

When doing *TST* calculations, every dip should have a deviation and every deviation should have a dip. The correct way to interpolate deviations is to interpolate the depths and coordinates from a deviation survey using the minimum curvature method of Taylor and Mason (1972). As in deviation interpolation, efforts to interpolate between dips by interpolating the dips and azimuths separately can introduce error. The interpolation method from Taylor and Mason (1972) can be adapted to interpolate dip angles as well.

Calculated dips derived from both formation image logs and from dipmeters are frequently accompanied by deviation data interpolated for each dip. Service companies will commonly interpolate the inclination and deviation azimuths separately. This is not the correct way to interpolate deviations. An example of how much error can be created by interpolating inclination and azimuth separately would be to take 2 deviation measurements, the first having inclination of 1° and deviation azimuth due East. The

second deviation has the same inclination, but deviation azimuth pointing due west. Averaging inclinations and azimuths separately yields 1° due South, when the true answer should be 0° inclination. The angular error is 1° , which may not sound large, but it is the same magnitude of the input inclinations. In other words, separately interpolating inclination and azimuth needlessly adds to calculation error. Since the interpolations are done for most dips, the cumulative effect over a given interval can be large.

Another common error in angular interpolation is to interpolate the direction vectors by separately interpolating the direction cosines. This will always come up with the wrong answer except when the angle to be interpolated is exactly halfway between the starting and ending angle. (This is vector average method can use a vector sum.) Again, the best way to interpolate between angles is by the minimum curvature method.

In a long interval where both dip and deviation change a great deal, it is possible to gain accuracy by subdividing the intervals and interpolating both dips and deviations as described above. This subdivision can result in higher accuracy because within each depth increment, dip and deviation will change over the larger interval, and a single-point calculation assumes that deviation angle and dip angle remain constant. In other words, each dip will have a different deviation, making the total *TST* different from when the whole interval is calculated. This also means that *TST* itself should not be interpolated linearly with depth if great precision is needed.

Averaging

When trying to refine dip data for *TST* calculation, averaging techniques may be used. There is problem in averaging dips that happens when operations are done on dip directions instead of dip poles. The problem is caused by the fact that a vector of the dip direction does not uniquely represent the dip plane. An example of this would be to take two dips, one 5° due East and one 5° due West. A vector average of the poles comes out with a correct dip of 0° . However, a vector average of the dips produces a vertical dip!

When averaging steep dips, care should be taken to decide when dips are overturned. Overturned dips can be represented by subtracting the dip from 180° and changing the azimuth by 180° . Alternatively, one can use eigenvector analysis and use the major pole for the average dip. This method will work on fractures as well as beds.

Spreadsheets

Programming the vectoral methods themselves into a spreadsheet is fairly straightforward. However, the recommended interpolations are somewhat more difficult (and tedious) in a spreadsheet, although not impossible. In Microsoft Excel, an alternative approach might be to write a VBA macro that takes input data columns as arrays and then writes output arrays back as columns.

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