THE EFFECT OF FRACTURE ORIENTATION AND HEIGHT ON FRACTURE FREQUENCY AND DENSITY

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ABSTRACT

The concepts of relative amplitude (*A*) and attenuation (α) are introduced to visualize the effect that borehole orientation has on fracture intersection frequency (*F*) in the borehole and on angular fracture frequency in polar plots. The shadow zone causes attenuation of frequency as the angle β between fracture planes and the borehole approaches zero. Shadow-zone polar plots are introduced to display the amount of attenuation in relation to angular fracture frequency and borehole orientation.

A general equation for *F* is extended to include all fractures, not just joints. That equation is used to derive another equation to convert borehole frequency data directly into density (P_{32}) using one free variable, effective fracture height (*H*). A method is given for determining *H* by counting fracture terminations. In addition, the general equation is extended to estimate joint width and length using fracture termination counts. Another method for determining *H* is given based on comparing directlycalculated P_{32} with frequency data. A third method is proposed for calculating *H* that is based on binning by of β values.

INTRODUCTION

Fracture density is important in determining reservoir properties such as permeability and porosity, which, in turn, help define reserves. Unfortunately, the frequency at which fractures are encountered in the borehole is not an accurate measure of fracture density in the surrounding rock.

Fracture Orientation

One variable that can affect fracture frequency is the relative orientation of fractures to the borehole. A simple example will be shown here to introduce the concepts and definitions. Figure 1 illustrates how fracture orientation can affect fracture frequency. In both cases, assume that the fractures are enclosed in a 1m by 1m box, while the borehole has a diameter of 0.2m. In both cases, the fracture density is the same at 10 fractures per square meter. Simply counting the number of fractures enclosed in the shaded, 1m portion of the borehole yields a fracture frequency of 10 fractures per meter on the left and 2 fractures per meter on the right. Clearly, fracture frequency does not always equal fracture density. A better way to calculate fracture density would be to add the fracture heights and divide that number by the area of the shaded portion of the borehole. (In sedimentary rocks, fracture size is generally referred to as fracture height.) The fracture density calculated on the left would be calculated as (0.2 m x 10)/(0.2 m x 1 m), while the fracture density calculated on the right would be calculated as $(1m \times 2)/(0.2m \times 1m)$. Both calculations yield the correct result of 10 fractures per meter. To extend the fracture density calculations to 3 dimensions, the fracture area is divided by the cube volume and gives identical results of 10 fractures per meter. Dimensionally, the units are the same, but the accuracy of results should increase as the measurement goes from fracture counts per linear meter to fracture length per area to fracture area per volume (Dershowitz and Herda, 1992).

Terzaghi (1965) was the first to derive a relationship of frequency *versus* fracture orientation. The paper correctly stated the problem for fractures in map view (the scan line), but her assertion that the same relationship held for boreholes was wrong. For instance, the scan line equation predicts that when fractures are parallel to the borehole, that no fractures will be encountered. It can clearly be seen from the right-hand side of Figure 1 that this is not the case. As will be demonstrated later in this paper, her derivation for boreholes left out two important variables, fracture height and borehole diameter. Although the exact formulation proved wrong, the idea that fracture frequency is dependent

on fracture orientation is essentially correct. In other words, as fractures approach parallel to the borehole, the fracture frequency will decrease.

Fracture Height and Borehole Size

In sedimentary rocks, fractures generally will have a rectangular shape. This is because fractures will propagate through rock until they hit a mechanical boundary (Narr and Suppe, 1991). The term the term "fracture height" refers to the smaller measurement. In fact, most derivations here and by other authors assume implicitly that fracture length is infinite.

In Figure 1, the borehole image or core on the left-hand side would give no indication of the fracture height (1m), while the on the right hand side, the fracture height would indeed be measurable. What is measurable in both cases is the enclosed fracture height, which was used in the density calculations above. Fracture height and effective fracture height are important in the derivation of the general frequency *versus* density relationship introduced later on in this paper. Another important consideration is the borehole diameter, because the interplay of borehole diameter with fracture height affects fracture density calculations.

Fracture Density

Dershowitz and Herda (1992) framed the dimensionality problem between fracture frequency and fracture density. (They used the term "intensity" instead of "density".) In their paper, they named simple fracture counts per unit length as P_{11} , fracture counts per unit area as P_{22} , and fracture counts per unit volume as P_{32} . As shown in Figure 1, even though the units the P_{11} and P_{22} results were the same at m⁻¹, the accuracy of the results increased with increasing dimensions of the measurement region. The same should be true going from P_{22} to P_{32} .

A great emphasis has been placed on P_{32} , because it is the intended measurement. The methods described herein are methods of removing sampling bias and/or converting frequency into P_{32} . In the

past, a common mistake has been to assume that bias-corrected frequency is equivalent to density.

Fracture Spacing

Studies explicitly dealing with *S* implicitly involve P_{32} . The main distinction between P_{32} and *S* is that *S*, by definition, assumes parallel fractures, while P_{32} has no assumption of orientation. That being said, it is useful to consider *S* in the non-orientation sense, in other words, the simple inverse of density. Used in this context, a good name for this might be "effective spacing".

Bed-Normal Fractures and Joints

Since most fractures are sub-perpendicular to bedding, these "bed-normal" fractures are important to this discussion. However, the subject of terminology for geological description of fractures has been a contentious one. It is common practice in field mapping that nearly all fractures are described as "joints". Because most fractures are bed-normal, the use of the term "joints" has evolved to specifically mean bed-normal fractures. Other definitions, such as the requirement that joints are tensional, have been added. Although the term "bed normal fracture" is sufficiently descriptive, using the word "joint" for a bed-normal fracture is much more concise and will thus be the term used in the rest of this paper.

A common feature of joints is that they are bed-bounded, that is, they terminate at bedding boundaries. Although joint height may depend on bed thickness, that does not necessarily mean that a given fracture will be entirely contained within a bed because joints do not necessarily terminate at every bed boundary. In other words, the bedding plane has to be a "mechanical" boundary in order for a joint to terminate there (Narr and Suppe, 1991).

Fractures in a Borehole

In reservoir characterization, a major goal is to measure the fracture porosity and permeability. Knowing these parameters should allow a comprehensive description of the effect of fractures on reservoir performance. When determining the amount of fracturing, the main problem is converting measurements into the three-dimensional (3D) measurement of fracture area per rock volume. Traditionally, collection of fracture orientation data consists of measuring dip, dip azimuth, and depth along a borehole. Additionally, there is usually a description, for example, "fracture", "open fracture", or "sealed fracture". Most of the time, bedding dip and description are collected along with the fracture data. Other common measurements can include quantities such the quality grade of the data or fracture aperture.

To determine fracture porosity, the fracture area per volume of rock is multiplied times fracture aperture or width. Fracture area can be derived from image logs, but the subject of determining fracture aperture is beyond the scope of this paper. It would suffice to say that fracture aperture is the hardest quantity to measure, and being equally as important as fracture area, it probably has the greatest potential for measurement error.

OBJECTIVES

Fracture data are often available only in tabular form with dip, dip azimuth, description, and aperture, but with no measure of the size of the fractures. In other words, only fracture frequency can be analyzed with such a data set. Correcting frequency data for shadow zone effects using scan line correction may be overly severe as β approaches zero. An equation will be developed here that takes partial fractures into account to convert fracture frequency into fracture density.

METHODS

Fracture Frequency and Relative Amplitude

The Scan Line Equation

Terzaghi (1965) introduced the concept of the "shadow zone" to describe how fractures are

encountered less frequently as their strike approaches that of a scan line. Figure 2 shows the basic relationships between fractures and a scan line on a map. Following is her equation for the relationships between fracture spacing and frequency, scan line length, and the angle between scan line and fracture strike:

$$\frac{N}{L_s} = \frac{\sin\beta}{S},\tag{1}$$

where:

N = Number of fractures intersected along the scan line,

 $L_S =$ scan-line length,

 β = minimum angle between the fracture strike and scan-line strike, and

S = fracture spacing in a direction perpendicular to the fracture set.

Assumptions implicit in equation 1 are that the scan line and fractures have no width and that the fracture length is infinite. These assumptions, especially regarding scan-line width and fracture length, will have significance later on when studying fractures encountered in a borehole.

Since the number of fractures divided by the length of the scan line is the fracture intersection frequency (F), equation 1 can be rewritten as

$$F = \frac{\sin\beta}{S}.$$
 (2)

Although the shadow zone concept is widely recognized, it is frequently overlooked, both quantitatively and qualitatively. This is especially true for polar plots, where gaps in coverage caused by the shadow zone are frequently misinterpreted as separation between fracture sets.

To convert this nomenclature to the case of a 3D borehole, β becomes the angle between the borehole and the fracture plane, the scan line becomes the borehole with dimensions of length (*L*_B) and diameter (*D*). Some authors have opted to use the angle measured between the normal to the fracture plane and the borehole (δ), instead of β . It is easy to convert between the two, because $\delta = 90^\circ - \beta$.

Useful identities when converting δ to β or the reverse are $\cos \delta = \sin \beta$ and $\sin \delta = \cos \beta$.

Terzaghi (1965) actually developed a relationship for boreholes, but it was essentially the same as the scan line equation. If one looks at the scan line equation, no fractures at all should be found in vertical wells with vertical fractures, but that is clearly wrong when one views a typical configuration (Figure 4). It is natural to assume that the reason for the problem is that the borehole diameter D is missing from the equation. One difficult aspect of attempting to include borehole diameter in a general relationship is that in many derivations D drops out and the scan line equation is left. The reason that these derivations fail is that they implicitly assume that the fractures are infinitely long, effectively turning the borehole into a line. Introducing a finite fracture height H allows derivation of a general equation that also includes D.

Scan Line Amplitude

A common presentation of fracture data is to plot fracture intersection frequency (*F*) versus depth. Correction for shadow zone effects should allow the display of relative proportions of fractures beyond the borehole. Figure 3 shows the bias correction (C_b) used to adjust scan line data for the shadow zone. (A common synonym for "bias correction" is "weighting".) To correct for bias, each fracture count is multiplied by $1/\sin\beta$. As β approaches zero, the correction approaches infinity. (Figure 2b is such an example.) In order to avoid huge over-corrections caused at small angles of β , Priest (1993) limited the correction size to a maximum value of 10, which is equivalent to β of about 5.7°. This method prevents the over-corrections when β is close zero, but does not get to the root of the problem.

Having very large bias corrections makes visualization of the problem difficult, so the variables relative amplitude (A) and attenuation (α) are introduced here in order to aid visualization. The relative amplitude is the amplitude at a given point relative to the amplitude at the maximum point, and attenuation is the decrease in relative amplitude relative to the maximum. To get a relative amplitude

whose value is 1 when $\beta = 90^{\circ}$, the amplitude at a point should be equal to the frequency at that point divided by the frequency at 90°, or

$$A = \frac{F}{F_{90}},\tag{3}$$

Where F_{90} is the frequency at $\beta = 90^{\circ}$. Substituting equation 2 into equation 3 for both F and F_{90} yields

$$A = \frac{\frac{\sin\beta}{S}}{\frac{1}{S}},\tag{4}$$

which simplifies to

$$A = \sin \beta \,. \tag{5}$$

Amplitude *A* is the inverse of the correction factor C_b and the attenuation α is equal to 1 - *A*. Figure 5 shows *A* versus β for the scan line equation (2). Now the left-hand side of the new plot is 0 instead of infinity, which is much more suitable for studying what happens in the common case where fractures are near parallel to the borehole.

The General Equation for Frequency

Other than the scan-line correction, no published method has been widely used for shadow zone correction of frequency-type borehole data. A potential candidate for shadow zone corrections was published in Narr (1996), his equation 15, but it was explicitly derived for joints, and the equation as published appeared to be dependent on the angle between the borehole and bedding, since the fractures being considered were joints. The original equation had a dependence on the angle of bedding relative to the borehole, but simplification causes the bedding angle to drop out. Following is Narr's general equation, algebraically simplified, for joints:

$$F = \frac{\sin\beta}{S} + \frac{D}{S \cdot H}.$$
(6)

Note that the left-hand term in the general equation (6) is identical to the scan line equation (2).

Although joints are the dominant fracture type, it is desirable to have a relationship valid for all fractures. Even though equation 7 has no dependence on bedding angle, since it was derived specifically for joints there is the possibility that it is valid only for joints. To demonstrate that the equation is valid for all fractures, it can be derived without using bedding orientation (see Appendix).

Estimating Effective Fracture Height

Figure 8 shows the general equation (6) plotted against the scan line equation (2). The righthand side of equation 6 is independent of β and is therefore constant. It is reasonable to assume that the excess fractures can be considered the partial fractures, since the fundamental difference between the general equation and the scan line equation is that the general equation considers partial fractures. At any angle of β , the ratio of partial fractures to whole fractures (*R*) should be the ratio of the righthand term of equation 6 divided by the left-hand term, which comes out to

$$R = \frac{D}{H\sin\beta}.$$
(8)

Another expression for R is the ratio of terminated fractures (N_T) to whole fractures (N_W) , or

$$R = \frac{N_T}{N_W} \,. \tag{9}$$

Combining equations 8 and 9 and simplifying yields

$$H = \frac{D \cdot N_W}{N_T \sin \beta}.$$
 (10)

The data can be binned by β value, perhaps at 10 degree intervals, and height calculated in the bins using the central β . Those heights can then be averaged for the final height. It is recommended that the bins be weighted according to the number of fractures in each bin, so as not to give disproportional weight to smaller samples. As β approaches zero in vertical wells with vertical joints, most fractures would be partial fractures and the actual fracture height (named "fracture width" below) can commonly be measured directly. Fracture length can then be found by equation 13 below, and

effective height can be found by equation 12 below. Effective height can also be found as described in the section on conversion of fracture frequency into fracture density.

Although equation 6 is fully 3D, it has the basic assumption that fractures have only one dimension, height. Since fractures essentially have two dimensions, this means that a truly general equation should account for two dimensions. In other words, *H* can actually be viewed as an effective height that is a composite of actual height (or width) and length.

Estimating Fracture Width and Length

A reasonable extension of equation 6 to include fracture width (W_F) and fracture length (L_F) would be

$$F = \frac{\sin\beta}{S} + \frac{D}{S \cdot W_F} + \frac{D}{S \cdot L_F} \,. \tag{11}$$

Since it is convenient to keep using effective fracture height (*H*), the following equation shows *H* in terms of L_F and W_F

$$H = \frac{L_F W_F}{L_F + W_F},\tag{12}$$

which can substituted into the general equation 6. (Equation 12 can be derived by combining equations 6 and equation 11 and solving for *H*.) Trying out equation 12, if $L_F = 10$ m and $W_F = 1$ m, then *H* would equal 0.90909m. It might seem counterintuitive that *H* would be smaller than the smallest dimension, but remember that the increase in fracture counts caused by L_F will appear to make *H* smaller. Modeling in the Appendix confirms equation 12, and, by induction, equation 11.

 L_F and W_F can be calculated from a known H, provided that the relative contributions of fracture terminations are known. Joints, in particular, are amenable to this type of calculation, because they are roughly rectangular in shape. If the number of bed-bounded terminations (T_W) and end terminations (T_L) are known, they should have an inverse relationship to W_F and L_F as follows:

$$\frac{T_W}{T_L} = \frac{L_F}{W_F} \,. \tag{13}$$

Solving equation 13 for L_F and substituting that into equation 12 we get

$$W_F = \frac{H\left(T_L + T_W\right)}{T_W} \tag{14}$$

to find W_F . Now, L_F can be found from a rearranged equation 8:

$$L_F = \frac{H \cdot W_F}{W_F - H} \,. \tag{15}$$

For example, consider a well in which all the fractures are joints. It has been determined from image logs that there are 100 bed-bounded terminations (T_W) and there are 10 end terminations (T_L). *H* has been calculated from equation 10 at 1.0m. From equations 14 and 15, $W_F = 1.1$ m and $L_F = 11$ m, respectively. Since end terminations are likely rare, a large sample interval would be needed to get a statistically valid result.

To give proper weighting to terminations, only a doubly terminated joint, that is a joint bounded above and below by bedding, should count as a full termination. If a joint is bounded above and below by bedding on one side, but it fails to appear again on the other side of the borehole, it should be considered both a bed termination and an end termination. The same is true for those cases in which a joint is terminated above and below by bedding on one side of the borehole but appears unbounded or only partly bounded by bedding on the other side. Where a fracture is bounded at both ends by either the top of the bed or by the bottom of the bed, this should count as a half termination.

The General Equation for Relative Amplitude

To translate equation 6 into relative amplitude, F is substituted twice into the general formula for amplitude (3) or

$$A = \frac{\frac{\sin\beta}{S} + \frac{D}{S \cdot H}}{\frac{1}{S} + \frac{D}{S \cdot H}}.$$
(16)

Rearrangement of equation 16 into results in

$$A = \frac{D + H\sin\beta}{D + H},\tag{17}$$

the general formula for relative amplitude. Attenuation and bias correction can be calculated as they were with the scan line equation, or $\alpha = 1 - A$ and $C_b = 1 / A$.

Shadow Zone Polar Plots

Figure 13 is an example of fracture pole plots in a horizontal well with the accompanying shadow zone calculated on the basis of borehole inclination. The shadow zone runs perpendicular to the borehole. In this case, the fractures are broadly distributed, making the shadow zone highly visible. In many cases, the fractures may be clustered away from the shadow zone, making the shadow zone much less apparent.

Figure 14 has the same plots as Figure 13, but in a vertical well with horizontal beds. In this case, the shadow zone is at the periphery of the plot, but before the angular frequencies drop off at the edges of the plots, they increase somewhat. This pattern is typical of a population of joints being sharply attenuated when the borehole is nearly vertical to bedding. Outside the borehole, there is a population of fractures that although present is being attenuated in the borehole itself. In this well, the permeability of the immediate borehole would be greatly limited, but in the rock surrounding the borehole, the fractures would still be common. In other words, a vertical well may be a poor way to drain a reservoir because of fracture frequency attenuation and not simply that a horizontal well would go through more formation and therefore more fractures.

Figure 15 is based on a published example from Barton and Moos (2010). It is clear that although the fracture patterns, at first glance, seem very different, they might actually be very similar.

Direct Calculation of Fracture Density

Direct ways exist for measuring fracture density. Dershowitz and Herda (1992) assert that the ultimate measure of fracture density is P_{32} or the "area of fractures per unit volume". To apply that concept directly to a borehole, one would calculate the area of each fracture and then divide the sum of the areas of the fractures by the volume enclosing the fractures. This will be called the "roundcylinder" method. An alternative method would be to extend the borehole to a square cylinder positioned with 2 of the sides parallel to the plane with in which the fracture normal and the borehole orientation lie. This will be called the "square-cylinder" method. Consider two slightly-inclined fractures intersecting a vertical borehole (Figure 9). Calculating fracture densities using fracture area versus volume would yield two different answers for two, identical, fully enclosed fractures. Figure 10 shows the fractures as seen from above. If the borehole is enclosed by a square cylinder aligned to each fracture, the area of overlap is the same and thus the calculated areas will be the same. Although there would seem to be a conflict between using a round-cylinder approach versus a square-cylinder approach, the two should give the same results on average, given a large enough sample size, because the lesser weight given to fractures approaching the edge of a circular cylinder is offset by dividing by the smaller volume of circular cylinder when compared to a square cylinder (see Appendix).

Narr, *et al.* (2006), have shown a P_{32} equation (3.2) based on division of enclosed fracture area to borehole volume. Since it is the most obvious method for direct calculation of P_{32} , it has probably been in use long before 2006, although no published reference has been found. It is probably the most widely used method for direct calculation of P_{32} . Although Narr, *et al.*, show the general approach, they do not go into detail about the mathematics.

Developing a P_{32} equation using the square-cylinder approach is straightforward. Figure 11 shows the two fractures displayed on left-hand side of the square cylinder in Figure 10 showing enclosed fracture height (*He*). To derive P_{32} , we need to sum the enclosed fracture areas and divide

that by the volume of the square borehole cylinder:

$$P_{32} = \frac{\sum_{i=1}^{N} Area_i}{Volume} = \frac{\sum_{i=1}^{N} He_i \cdot D}{L_B \cdot D \cdot D} = \frac{\sum_{i=1}^{N} He_i}{L_B \cdot D}.$$
(18)

Shown in terms of fracture spacing, equation 18 is inverted to become

$$S = \frac{L_B \cdot D}{\sum_{i=1}^{N} He_i}.$$
(19)

Equation 19 is identical to Narr (1996) equation 5, except that his fracture height was the height of a vertical joint in a vertical borehole ($\beta = 0$). The reason that the equations are identical is that in the specific case of a vertical borehole with vertical fractures, enclosed fracture height and actual fracture height are the same thing. In fact, Narr (personal comm.) has used equation 19 to calculate *S*, even with $\beta > 0$, by using *He* in place of *H*.

Although counterintuitive, an equation that uses fracture height divided by area, seemingly P_{21} , is as at least as good as fracture area to borehole volume calculations. Equation 18, however, was actually derived using area per volume, but the extra dimensions canceled out, so it is not simply just a P_{21} equation. An additional insight is that Dershowitz and Herda (1992) actually define P_{21} as the "number of fractures per unit area of trace plane", because they were considering lines on a map, not lines on a cylinder. Lines on a map are much less constrained than inscribed traces on a cylinder. It is therefore not much of a stretch to expand the traces on a round cylinder to the edges of a square cylinder. Not much extrapolation has occurred, but the solution is at least as accurate as the direct ellipsoidal area to cylindrical volume solution (see Appendix).

Although the square-cylinder and the round-cylinder methods are equivalent over sufficiently long intervals, the square-cylinder method can be advantageous, because similar fractures are given similar weightings. For example, in a vertical well with vertical joints of the same height, individual P_{32} calculations will vary widely for when using the round-cylinder method, because, for instance, a

fracture just glancing the side of the borehole will have a very small area compared to a fracture through the center of the borehole. In contrast, the square-cylinder method will show the fractures as being equivalent. In other words, in the round-cylinder method, variability is introduced simply because of a fracture's position relative to the borehole and not because of any intrinsic fracture properties.

A third possibility for the direct calculation of P_{32} is dividing fracture trace length to borehole surface area (Wang, 2005). This will be called the "surface trace to area" method. This method would have an advantage over the other two methods in that there would be no extrapolation, neither inward nor outward from the borehole surface. This would also make data collection easier, because no interpretation would have to be made as to whether two traces might belong to the same fracture as in the other two approaches. However, tests reveal that it is probably not equivalent to the other two (see Appendix).

Conversion of Fracture Frequency into Fracture Density

As described above, P_{32} can be measured directly from image logs. In practice, however, the data are commonly collected in such a manner that this cannot usually be done. This is because each fracture is assigned a dip, azimuth, quality, etc., but no measure is given that would enable direct calculation of fracture density, specifically a number indicating the partial amount of a fracture present in the borehole.

To derive the conversion of *F* into P_{32} , first we put the general frequency equation (6) into units of fracture density by multiplying both sides by *He/D*:

$$\frac{N \cdot He}{L_{\rm P}D} = \frac{\sin\beta \cdot He}{S \cdot D} + \frac{He}{H \cdot S}.$$
(20)

From equation 18, the left-hand side of equation 20 is P_{32} , so the equation for P_{32} is:

$$P_{32} = \frac{\sin\beta \cdot He}{S \cdot D} + \frac{He}{H \cdot S} .$$
(21)

From the appendix,

$$He = \frac{D \cdot H}{D + H \sin \beta} \,. \tag{22}$$

Substituting equation 22 into equation 21, dividing the resulting equation by equation 6, and then simplifying yields P_{32} in terms of *F*, *D*, *H*, and β :

$$P_{32} = \frac{F \cdot H}{D + H \sin \beta} \,. \tag{23}$$

If equation 6 is substituted for F in equation 23, simplification results in the expected $P_{32} = 1/S$.

Figure 12 shows the results of using equation 23 to calculate P_{32} from *F*. As expected, for all points on the graph, P_{32} calculated from *F* yields an answer of 0.5m^{-1} , which in the example, is the inverse of the spacing of 2.0m. When applying equation 23 to well data, at each sample point P_{32} will not necessarily be correct, but the overall P_{32} should be a good approximation over a long interval, assuming that the proper *H* is used. If only fracture counts are available (frequency data), then equation 23 is suitable for calculating fracture density. However, when the original image data are available, it is recommended that P_{32} be calculated directly as described in the section above on direct calculation of P_{32} . Equation 23 should also provide a means for comparing frequency datasets to directly-calculated datasets, which, in turn, should allow calculation of *H* independent of fracture termination counts.

At this point it should be noted that *F* plotted on logs often uses overlapping counting windows, otherwise the data would be too sparse to interpret qualitatively. In directly-measured P_{32} , this overlap can be simulated by a running average. For example, if the counting window is 10m and the sample interval is 1m in a converted frequency log, the directly-measured P_{32} data sampled on a 1m interval can be smoothed with a 10-point running average. (It is important to note here that to compare the *F* data to the smoothed P_{32} data, the *F* data should be normalized, or divided by 10 in this example.)

Figure 16 shows an uncorrected frequency curve versus a weighted curve. Although the windowing used in calculation smoothes the effect of the weighting somewhat, the effect of the weighting varies substantially within the small interval shown. Figure 16 also shows normalized, bias-corrected frequency and density together. It is clear that the density is always lower than the frequency, which makes sense, because the frequency curve gives the same weight to all fractures, while the density curve gives the partial fractures less weight. It is also clear that bias-corrected frequency curve gives the regarded quantitatively as density.

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APPENDIX

Deriving a General Equation for Frequency

Consider a vertical well with jointed, horizontal beds (for example, Figure 4)—a common configuration and at the same time it is a good example for shadow-zone attenuation of fractures in a borehole. According to the scan line equation 1, fractures should be extremely rare or nonexistent, but it is clear that this is not true. An equation for this configuration is a good starting point for building a general equation. Below is Narr (1996) equation 5 rewritten in non-probabilistic terms:

$$S = \frac{D \cdot L}{N \cdot H},\tag{A-1}$$

where *H* is the fracture height and *D* is the borehole diameter. The borehole and joints were assumed vertical as in Figure 4. In other words, equation A-1 represents the case where β is equal to 0. Rearranging equation A-1 gives the expression for the general equation for fracture counts at $\beta = 0$:

$$N_0 = \frac{D \cdot L}{S \cdot H}.$$
 (A-2)

Figure A-1B shows a borehole traversing fractures of equal height (H) that have been rearranged in a vertical band perpendicular to the borehole. Bands of this type should be able to represent any arrangement of fractures with the same spacing. In other words, as long as the areal distribution is the same, the fracture density remains the same. Starting with N_0 , we can build an equation by correcting for changes in fracture height components and apparent fracture spacing:

$$N = N_0 \frac{H_{\parallel}}{H} \frac{S}{S_{\perp}} \left(1 + \frac{H_{\perp}}{D} \right), \tag{A-3}$$

where H^{\perp} is the fracture height projected perpendicular to the borehole, H_{\parallel} is the fracture height projected parallel to the borehole, and S^{\perp} is the apparent spacing perpendicular to the borehole. The H_{\parallel} correction accounts for the changing width of the bands, the S^{\perp} correction accounts for the wider apparent spacing of the fractures, and the H^{\perp} correction accounts for excess fractures encountered as more of each fracture is presented to the borehole. (In looking at Figure A-1, perhaps the least obvious expression in equation A- is the expression " $1 + \frac{H_{\perp}}{D}$ ". The testing below will confirm that expression.) Substituting equation A-2 and trigonometric relationships for their respective variables into equation A-3 yields

$$N = \frac{D \cdot L}{S \cdot H} \frac{1}{\cos \beta} \cos \beta \left(1 + \frac{H \sin \beta}{D} \right).$$
(A-4)

Simplification and substitution of F for N/L_B in A-4 yields

$$F = \frac{\sin\beta}{S} + \frac{D}{S \cdot H},\tag{A-5}$$

the general equation for fracture frequency (equation 6 in the main text and Narr, 1996, equation 15.) It is important conceptual point that equation A-5 counts partial fractures the same as whole fractures. This is especially relevant when converting frequency into density and also the testing below.

Testing Equations at $\beta = 90^{\circ}$

The general frequency equation A-5, when $\beta = 90^{\circ}$, reduces to

$$F = \frac{1}{S} \left(1 + \frac{D}{H} \right). \tag{A-6}$$

This configuration would commonly be found in a horizontal well drilling through horizontal, jointed beds. In terms of fracture counts, equation A-6 is

$$N = \frac{L_B}{S} \left(1 + \frac{D}{H} \right), \tag{A-7}$$

where L_B is the length of the borehole interval. Modeling joints in this way might be difficult, however, because under these strict conditions, the borehole would either be straddling a bedding plane or entirely within beds (Figure A-2). Therefore, the modeling was developed with the idea that succeeding joints can be oriented randomly with respect to each other. As a further aid in modeling, the fractures can be visualized as being arranged in a continuous surface, edge to edge. As long as the proper spacing (density) is maintained, the general equation should still apply, since arrangement of fractures is not important. Each successive sheet of fractures, although parallel, is placed randomly with respect to the others.

Testing the General Equation

The expression "D/H" in equation A-7 states the probability of hitting a partial fracture for each whole fracture encountered. To test the veracity of the general equation in a probabilistic manner, a simple model was constructed using that expression. A random number was generated between 0 and the *H*. If that number was greater than *D*, then a whole fracture was encountered. If that number was less than *D*, then a partial fracture was encountered. Figure A-3 shows an example of the modeling. The probability of hitting partial fractures predicted using equation A-7 is very close to the modeled probability. The same approach will be used for testing equation 12, the relationship between *H* and fracture width (W_F) and fracture length (L_F).

Testing Fracture Width (W_F) and Length (L_F)

In a manner similar to that for H, a number was chosen randomly between 0 and W_F and a number was chosen randomly between 0 and L_F . If either number was less than D, then the fracture count was increased by 1. If both were less than D, then the count was increased by 2. Example results are shown in Figure A-4. It was found that the mean effective fracture height (H) was comparable to that predicted by effective fracture height equation (12).

Testing the Equivalence of Round-Cylinder versus Square-Cylinder P₃₂

It is a logical assumption that round-cylinder and square-cylinder methods of calculating P_{32} are equivalent, because the smaller areas encountered in the round-cylinder method are compensated by dividing by a smaller volume. However, since the square-cylinder method always views the fractures

in the same aspect, that is, parallel to a plane in which the fracture pole and the borehole direction lie, there could be the possibility that the two methods would differ. The modeling was undertaken to rule that possibility out.

The most severe differences between the two methods should be in the case where a vertical well is intersecting vertical joints, because in the round-cylinder method would give small areas for fractures that barely intersect the edge of the borehole compared to large areas when fractures intersect at the center of the borehole. On the other hand, the square-cylinder method will always yield the same answer given the same fracture height.

Two basic methods were used to do the modeling, one in which fracture planes with random strike were generated at random coordinates and another in which perpendicular fracture planes were placed at constant small intervals along an axis (Figure A-5). To simplify the calculations, borehole length (L_B) was set equal to H. Following are the equations used in the calculations:

1. Square cylinder =
$$\frac{1}{D} = \frac{H \cdot D}{L_B D^2}$$

2. Round cylinder =
$$\frac{c}{\pi r^2} = \frac{H \cdot c}{\pi r^2 L_B}$$

3. Surface trace to Area = $\frac{c+H}{\pi r(r+H)} = \frac{2c+2H}{2\pi r(r+L_B)}$,

where c = chord length and <math>r = D/2. The full expressions are shown on the right for reference. The random-strike and constant-interval methods yielded the same answers to 4 significant digits after it was found that the starting points on the random-strike method had to extend far from the borehole to get an even distribution. The final calculations used an area for generating the random starting points that extended 5 times the borehole diameter from the center. Therefore, the random-strike calculations took much longer, since large majority of the random lines did not intersect the borehole, and calculations were not performed if they did not intersect it. All of the calculations were performed

1,000,000 times to get the average modeled values, although it was really not necessary for the squarecylinder method since 1/D is constant.

The modeling demonstrated the equivalence of the square-cylinder method to the roundcylinder method. Surprisingly, the surface trace to area method gave different answers than the other two. For example, with *D* of 0.25m and *H* of 1.0m, the square-cylinder and the round-cylinder methods gave an average P_{32} of 4.0m⁻¹, while the surface trace to area method yielded 2.7080m⁻¹. The differences in the trace to area method with the other methods do not imply a simple linear or constant relationship with changing *D*, but a more complex relationship cannot be ruled out.

Calculating Enclosed Fracture Height

Enclosed fracture height *He* differs from effective fracture height *H* in that it is measured only within the confines the borehole rectangular cylinder (see Figure 11 in the main text). In order to derive an equation for converting P_{32} to *F*, it is necessary to define an average expected *He* in terms of *F*, *H*, and β . Figure A-6 shows a series of closely spaced fractures, all with the same *H*. On the lefthand side, fractures are arranged such that the minimum number of full fractures covers the entire borehole. On the right-hand side the fractures have been rearranged in a rectangular area. The trapezoidal area on the left is equal to the rectangular area on the right. If the rectangular area on the right represents full fractures and the enclosed area within the borehole represents the partial fractures, then the ratio of the full area to the ratio of the enclosed area should be equal to the ratio of *H* to *He*, or

$$\frac{A_f}{A_e} = \frac{H}{He},\tag{A-8}$$

where A_f is the area of the full fractures and A_e is the area of the enclosed fractures. Substitution of appropriate expressions for the areas yields

$$\frac{H_{\parallel}(D+H_{\perp})}{H_{\parallel}D} = \frac{H}{He}, \qquad (A-9)$$

Which, after $Hsin\beta$ is substituted for H[⊥], simplifies to

$$He = \frac{D \cdot H}{D + H \sin \beta} \,. \tag{A-10}$$

FIGURES



Figure 1. A set of 1m-long fractures intersecting perpendicular to (left) and parallel to (right) the borehole. (The fractures are assumed to extend very far both toward and away from the observer, and the fracture spacing is small to aid in visualization.) Although the fracture density is identical in both cases, the fracture frequency on the right is 1/5 of the frequency on the left.



Figure 2. A) Map view of the relationship between a set of fractures and a scan line. B) The same fracture set, but with the scan line is parallel to fracture strike. Since the width of the scan line and the fractures is undefined (infinitely small), the fracture intersection frequency (F) is zero.



Figure 3. The bias correction (C_b) derived from the scan line equation (1). Priest (1993) suggested a limit of 10 for the bias correction, which is equivalent to β of about 5.7°. A line has been drawn at $C_b = 1$ in order to show that C_b approaches 1 as β approaches 90°. (For the scan line equation (2), the bias correction is $C_b = 1 / \sin \beta$.)



Figure 4. An example of a vertical well with vertical fractures. The fractures are assumed perpendicular the plane of section. In this example, it is clear that fractures will be encountered fairly often. This fact directly contradicts the scan line equation, which predicts that no fractures should be encountered when fractures are parallel to the borehole.



Figure 5. Relative amplitude (*A*) plotted against β . The shadow zone can be viewed as the attenuation (α) of the relative amplitude caused by the angle that the scan line makes with the fracture. Quantitatively, $C_b = 1/A$ and $\alpha = 1-A$



Figure 6. The general equation 6 amplitude relative to Terzaghi (1965) and Priest (1993) amplitudes. In this case D = 0.25m and H = 1.5m ($A_0 = 0.143$). There is a significant difference in amplitude, even with the relatively low value for A_0 .



Figure 7. Same as Figure 6, but in terms of bias correction. In this case, the intercept for Priest's ceiling and the general equation are the same. At $\beta = 5^{\circ}$, the difference between Priest's ceiling of 10 and the general equation is significant at about 54%.



Figure 8. The general equation (6) plotted against the scan line equation (2). Since the left-hand term of the general equation is equivalent to the scan line equation, the difference between the two curves shows the excess fractures, which should also be partial fractures. In this case, the borehole diameter is 0.25m, the fracture height is 1m and the spacing is 1m. The right-hand side of the general equation (the excess fractures) is independent of β and is a constant 0.25m⁻¹.



Figure 9. Two identical, slightly inclined fractures intersecting a vertical borehole. A) A fracture intersecting the borehole close to the center line of the borehole. B) A fracture intersecting the borehole with the lower edge close to the side of the borehole. Although the height and inclination of the fractures are identical, the included areas are different. If P_{32} were to be calculated as the enclosed fracture area per unit volume, then these two, identical, totally enclosed fractures would yield different values of P_{32} .



Figure 10. The same two fractures as in Figure 9 viewed from the top. On the left, the overlap shows the differences in area for round-cylinder calculation of fracture density. On the right, the use of a square cylinder to enclose the borehole gives the two fractures the same area of overlap.



Figure 11. The two fractures, A and B, in Figure 9 seen together on the left-hand side of the rectangular borehole cylinder. Fracture C has been added on the left of the cylinder to show how enclosed fracture (*He*) height is measured when a fracture intersects a side.

Converting Frequency to Density



Figure 12. Comparing fracture frequency (F) to density (P_{32}) and bias-corrected frequency. Equation 23 has been used to convert F at each point to P_{32} . When applied to actual well data, each density point will effectively be an average expected value and thus will not compare exactly to a directly calculated P_{32} . Overall, though, the results should compare provided that the proper fracture height (H) has been used. It might be possible to compare these calculations to direct calculations in order to derive H without having to depend on counting fracture terminations. Note that bias-corrected frequency and density are different, because frequency counts all fractures the same and density takes partial fractures into account.



Figure 13. Polar plots illustrating a shadow zone in a horizontal well. The left plot is a shaded contour plot of fracture-pole frequency within 18° of each grid point. On the right is a plot of the individual fracture poles on top of a shadow zone attenuation based on the borehole orientation within the interval. It is a common misconception in polar plots that this pattern would represent two broad fracture sets. Instead, the apparent absence of poles in the NE/SW direction is caused by the shadow zone rather than a separation between fracture sets. (Polar plots in this paper are in stereographic projection and upper hemisphere unless otherwise stated.)



Figure 14. Polar plots illustrating a shadow zone in a near-vertical well. The plots are the same as in the horizontal example (Figure 13). The pattern is typical of vertical wells in horizontal beds. The joints, which are near perpendicular to bedding, start to increase toward the edge of the plot and rapidly fall off as the joints become nearly parallel to the borehole. This would imply that there is a much larger fracture population in the country rock. Undoubtedly, the low number of fractures in this particular borehole would justifiably restrict its calculated fracture porosity and permeability, but the porosity and permeability in this borehole should not be blindly extrapolated out into the formation.



Figure 15. Figure 20 from Barton and Moos (2010). The upper polar plots are the original figure. It consists of two, lower hemisphere, fracture frequency plots from adjacent boreholes. In the middle plots, the shadow zones have been calculated using borehole deviation that has been estimated using azimuths from the figure and inclinations based on apparent attenuation in the plots. In the lower plots, the shadow zones have been switched, with the Well A shadow zone plotted over Well B, and the Well B shadow zone plotted over well A. It is apparent that much of the dissimilarity of the plots is caused by having different shadow zones and not necessarily having different fracture sets.





Figure 16. Fracture frequency and density curves with dips and borehole deviations for part of the horizontal well in Figure 13. In the density and corrected frequency curves, H = 0.8m and D = 20cm. The curves are all sampled at 1 m, but the counting window is 10 m. The difference in the two curves on the left shows the differences in raw frequency to bias-corrected frequency caused by changing β . The right-hand curves show normalized density (P_{32}) versus normalized, bias-corrected frequency (F). They are different because frequency counts partial fractures the same as whole fractures. This is why the bias-corrected fracture frequency should never be used as fracture density.



Figure A-1. Arrangement of fractures for the derivation of a general equation. A) A typical arrangement of fractures at $\beta = 0$. (For illustration purposes, spacing is much smaller than is typical.) This configuration is common in vertical wells with horizontal beds. B) As β increases, the fractures can be rearranged in vertical bands, and as long as the fracture height and spacing remain the same, the fracture density should remain constant. C) Changes in fracture height components, H^{\perp} and H^{\parallel} and S^{\perp} can be used to derive the general equation. The change in fracture counts N with β can be calculated relative to the fracture counts at $\beta = 0$. (Note that both fracture height H and spacing S remain constant in both arrangements.)



Figure A-2. Graphical example for analyzing equation A-7. If S = 1.0, H = 1.0, D = 0.25, and $L_B = 4$, then 4 fractures would be expected most of the time, except when a bed boundary was encountered, at which time 8 fractures would be expected. It appears reasonable that bed boundaries would be encountered about a quarter of the time, since the borehole is a quarter of the fracture height, therefore the value of 5.0 fractures predicted by equation A-7 would seem reasonable. The modeling was undertaken to confirm that idea probabilistically.

H = 0.5m, D = 0.2m, n = 128



Figure A-3. Summed distribution of frequency of 100,000 runs over 128 trials (*N*) for H = 0.5m, D = 0.2m, S = 1 and $L_b = 128$. The multiple runs were done to get a smooth curve. The diamonds represent the actual summed values, while the solid line represents values predicted for a binomial distribution. The predicted probability (D / H, equation A-6) was 0.4 and the mean probability was 0.3999, which relates to a predicted F of 1.4 and a calculated F of 1.3999. The chi square test yielded 0.34814, which is good but not nearly a perfect 1.0. Oddly enough, the values of the chi square test got smaller as more runs were summed. Perhaps a better measure of the fit of the two curves is the correlation coefficient of 0.9998. The way to read an approximate probability directly from the chart is to take the maximum value of k (the number of "successes" or partial fractures) and divide it by 128 (the number of "trials" or fractures hit).

Wf = 1m, Lf=10m, D = 0.2m, n = 128

Figure A-4. As with Figure A-3, but for testing fracture width (W_F) and fracture length (L_F). $W_F = 1.0$ m, $L_F = 10$ m, and D = 0.2m. Effective fracture height (H) as predicted by equation 12 was 0.90909m, and the mean effective fracture height from the model was 0.90908m. The correlation coefficient between the actual and the binomial-predicted frequency was 0.9998.

Figure A-5. A.) A view from above showing a randomly generated, vertical fracture. Fractures were generated at random points within and surrounding the borehole and the fracture strike was generated at random angles. B.) A view from above showing how fractures were generated at equal intervals along an axis. Both of these models showed the equivalence of the square-cylinder method and the round-cylinder method, but surprisingly, the surface trace to area method did not agree with the other two.

Figure A-6. Calculation of enclosed fracture height (He). The spacing shown is small to aid in visualization. The group of whole fractures on the left has been rearranged on the right to show the relative area of excess fractures versus the area of the enclosed fractures.